N, n, dimensionless complexes introduced into (5); v, dimensionless volume; ν, parameter introduced into (5); z_*, dimensionless bed height in steady fluidized state; ω, circular frequency.

LITERATURE CITED

1. V. A. Borodulya, Yu. A. Buevich, and V. V. Zav'yalov, Inzh.-Fiz. Zh., 31, No. 3 (1976).

EXACT SOLUTION OF COMBINED HEAT- AND MASS-TRANSFER PROBLEM DURING FILM ABSORPTION

N. I. Grigor'eva and V. E. Nakoryakov

UDC 536.248.2

Exact solutions of the system of equations of heat and mass transfer accompanying absorption of vapor by a liquid film are obtained. Expressions for the main characteristics of heat and mass transfer are obtained.

Numerous processes used in chemistry, refrigeration, etc. entail the absorption of vapor by a liquid solution. A characteristic feature of such processes is the combined transfer of heat and absorbate in the liquid. In practical engineering calculations, however, heat- and mass-transfer processes are usually considered separately.

In the present paper we use a simple model to investigate the mutual effect of heat transfer and diffusion processes during absorption by a film.

The treatment of the problem of combined heat and mass transfer during absorption of a pure (with no admixture of gas) vapor by a film of solution flowing down a vertical wall is based on the following assumptions:

1) the wall is isothermal and impermeable for the absorbed substance;

2) the film thickness δ is constant;

3) the flow of liquid is laminar;

4) at the liquid–vapor interface the "absorbate–liquid solution" system is in a state of saturation;

5) wave processes in the liquid do not affect heat or mass transfer;

6) all the physical parameters of the problem (thermal diffusivity, diffusion coefficient etc.) are constant in the considered ranges of temperature and pressure.

As a model representing the state of saturation we select a linear relation between the concentration and temperature

$$\bar{C} = d\bar{T} + b.$$

The coefficients d and b are determined by the vapor pressure. We introduce a Cartesian coordinate system (x', y'), whose x' axis coincides in direction with the velocity v of liquid in the film and whose coordinate origin lies on the solid wall. We assume that in the cross section x' = 0 the liquid temperature T_0 and concentration C_0 are constant over the cross section, and C_0 is less than the saturation value corresponding to temperature T_0, i.e., $C_0 < dT_0 + b$.

We solve the problem on the assumption that v = const. In dimensionless form the system of equations representing heat and mass transfer in the film and the boundary conditions are as follows:

Institute of Thermophysics, Siberian Branch of the Academy of Sciences of the USSR. Special Design Office for Power and Chemical Plant Machinery, Novosibirsk.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 33, No. 5, pp. 893-898, November, 1977. Original article submitted October 28, 1976.

0022-0841/77/3305-1349$07.50

$$\begin{cases} \dfrac{\partial T}{\partial x} = \dfrac{\partial^2 T}{\partial y^2}; \\ \dfrac{\partial C}{\partial x} = \dfrac{1}{\mathrm{Le}} \dfrac{\partial^2 C}{\partial y^2}; \end{cases} \tag{1}$$

$$T(0, y) = M_1 = 1 - \frac{T_w}{T_0}; \tag{2}$$

$$C(0, y) = M_2 = 1 - \frac{dT_w + b}{C_0}; \tag{3}$$

$$T(x, 0) = 0; \tag{4}$$

$$\left.\frac{\partial C}{\partial y}\right|_{y=0} = 0; \tag{5}$$

$$C(x, 1) = \frac{dT_0}{C_0} T(x, 1); \tag{6}$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=1} = \frac{\mathrm{Ka}}{\mathrm{Le}} \cdot \frac{C_0}{dT_0} \left.\frac{\partial C}{\partial y}\right|_{y=1}. \tag{7}$$

Here $\mathrm{Ka} = r_a d/c_p$ – a dimensionless complex – is an analog of the **phase-transition criterion,** since 1/d has the dimension of temperature.

Condition (7) expresses the fact that all the heat released by absorption is expended on heating of the liquid by conduction.

The system of equations (1) with boundary conditions (2)-(7) is solved by the Fourier method; i.e., the solutions are represented in the form of expansions in series of eigenfunctions

$$T = \sum_n A_n X_n^{(1)}(x) Y_n^{(1)}(y), \tag{8}$$

$$C = \sum_n B_n X_n^{(2)}(x) Y_n^{(2)}(y). \tag{9}$$

From the system of equations (1) and boundary conditions (4) and (5) we obtain

$$Y_n^{(1)}(y) = \sin(k_n y); \quad Y_n^{(2)}(y) = \cos(\sqrt{\mathrm{Le}}\, k_n y), \tag{10}$$

$$X_n^{(1)}(x) = X_n^{(2)}(x) = \exp(-k_n^2 x), \tag{11}$$

where k_n are the eigenvalues.

Conditions (6) and (7) give the following equalities:

$$\frac{B_i}{A_i} = \frac{dT_0}{C_0} \frac{Y_i^{(1)}(1)}{Y_i^{(2)}(1)}, \tag{12}$$

$$\frac{B_i}{A_i} = \frac{\mathrm{Le}}{\mathrm{Ka}} \frac{dT_0}{C_0} \frac{Y_i'^{(1)}(1)}{Y_i'^{(2)}(1)}, \tag{13}$$

from which we obtain the transcendental equation for the eigenvalues k_n

$$\frac{Y_n^{(1)}(1)\, Y_n'^{(2)}(1)}{Y_n^{(2)}(1)\, Y_n'^{(1)}(1)} = \frac{\mathrm{Le}}{\mathrm{Ka}}$$

or

$$f(k_n) = \mathrm{tg}(k_n)\, \mathrm{tg}(\sqrt{\mathrm{Le}}\, k_n) + \frac{\sqrt{\mathrm{Lu}}}{\mathrm{Ka}} = 0. \tag{14}$$

To calculate the expansion coefficients A_n and B_n we obtain the orthogonality relation for the eigenfunctions.

Since we have the following equations for $Y_i^{(1)}$ and $Y_j^{(1)}$

$$Y_i''^{(1)} + k_i^2 Y_i^{(1)} = 0, \quad Y_j''^{(1)} + k_j^2 Y_j^{(1)} = 0 \ (i \neq j),$$

by multiplying them respectively by $Y_j^{(1)}$ and $Y_i^{(1)}$, subtracting the obtained equalities from one another, and integrating with respect to y from 0 to 1, using the condition $Y_{i,j}^{(1)}(0)=0$ [condition (4)], we obtain the following equality:

$$(k_i-k_j)\int_0^1 Y_i^{(1)}Y_j^{(1)}dy = Y_j^{\prime(1)}(1)\,Y_i^{(1)}(1)-Y_i^{\prime(1)}(1)\,Y_j^{(1)}(1). \tag{15}$$

Similarly, using condition $Y_{i,j}^{\prime(2)}(0)=0$ [condition (5)], we obtain

$$\mathrm{Lu}\,(k_i-k_j)\int_0^1 Y_i^{(2)}Y_j^{(2)}dy = Y_j^{\prime(2)}(1)\,Y_i^{(2)}(1)-Y_i^{\prime(2)}(1)\,Y_j^{(2)}(1). \tag{16}$$

We multiply the left-hand side of (16) by

$$\frac{B_i}{A_i}\,\frac{B_j}{A_j}\,\frac{C_0}{dT_0}\,\frac{\mathrm{Ka}}{\mathrm{Lu}}\,\frac{C_0}{dT_0}.$$

The equality still holds if, using relations (12) and (13), we multiply the first term on the right-hand side by

$\dfrac{Y_j^{\prime(1)}(1)}{Y_j^{\prime(2)}(1)}\,\dfrac{Y_i^{(1)}(1)}{Y_i^{(2)}(1)}$ and the second term by $\dfrac{Y_i^{\prime(1)}(1)}{Y_i^{\prime(2)}(1)}\,\dfrac{Y_j^{(1)}(1)}{Y_j^{(2)}(1)}$:

$$\frac{B_i}{A_i}\,\frac{B_j}{A_j}\,\frac{C_0}{dT_0}\,\mathrm{Ka}\,\frac{C_0}{dT_0}(k_i-k_j)\int_0^1 Y_i^{(2)}Y_j^{(2)}dy = Y_j^{\prime(1)}(1)\,Y_i^{(1)}(1)\;Y_i^{\prime(1)}(1)\,Y_j^{(1)}(1). \tag{17}$$

Comparing (15) and (17), we obtain the required orthogonality relation in the form

$$A_iA_j\int_0^1 Y_i^{(1)}Y_j^{(1)}dy - B_iB_j\,\mathrm{Ka}\,\frac{C_0^2}{d^2T_0^2}\int_0^1 Y_i^{(2)}Y_j^{(2)}dy = 0. \tag{18}$$

From the initial conditions (2) and (3) and relation (18) we derive the following expressions for A_n and B_n:

$$A_n = \frac{M_1\int_0^1 Y_n^{(1)}dy - M_2\,\mathrm{Ka}\,\dfrac{C_0}{dT_0}\,\dfrac{Y_n^{(1)}(1)}{Y_n^{(2)}(1)}\int_0^1 Y_n^{(2)}dy}{\int_0^1 Y_n^{2(1)}dy - \mathrm{Ka}\,\dfrac{Y_n^{2(1)}(1)}{Y_n^{2(2)}(1)}\int_0^1 Y_n^{2(2)}dy},$$

$$B_n = \frac{dT_0}{C_0}\,\frac{Y_n^{(1)}(1)}{Y_n^{(2)}(1)}\,A_n.$$

After calculation of the integrals, using expressions (10) we finally obtain

$$A_n = \left\{4\left[\left(1-\frac{T_W}{T_0}\right)(1-\cos k_n)-\left(1-\frac{dT_W+b}{C_0}\right)\times\right.\right.$$

$$\left.\left.\times\frac{\mathrm{Ka}}{\sqrt{\mathrm{Le}}}\,\frac{C_0}{dT_0}\sin k_n\,\mathrm{tg}\sqrt{\mathrm{Le}}\,k_n\right]\right\}\Big/\left\{(2k_n-\sin 2k_n)-\frac{\mathrm{Ka}}{\sqrt{\mathrm{Le}}}\,\frac{\sin^2 k_n}{\cos^2\sqrt{\mathrm{Le}}\,k_n}(2\sqrt{\mathrm{Le}}\,k_n+\sin 2\sqrt{\mathrm{Le}}\,k_n)\right\}, \tag{19}$$

$$B_n = \frac{dT_0}{C_0}\,\frac{\sin k_n}{\cos\sqrt{\mathrm{Le}}\,k_n}\,A_n. \tag{20}$$

The solutions obtained in this way enable us to calculate all the characteristics of the heat- and mass-transfer process: the heat and mass fluxes through the film surface, the heat flux through the solid wall, and the temperature and concentration averaged over the cross section

$$Q_{hs} = \frac{g_h\delta}{\lambda T_0}\bigg|_{y=\delta} = \sum_n A_n\exp(-k_n^2x)\,Y_n^{\prime(1)}(1)\;\sum_n A_n\exp(-k_n^2x)\,k_n\cos k_n; \tag{21}$$

$$Q_{hw} = \frac{g_h\delta}{\lambda T_0}\bigg|_{y=0} = \sum_n A_n\exp(-k_n^2x)\,Y_n^{\prime(1)}(0)\sum_n A_n\exp(-k_n^2x)\,k_n; \tag{22}$$

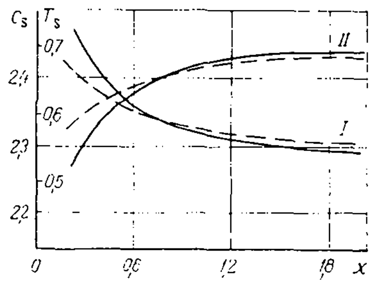

Fig. 1

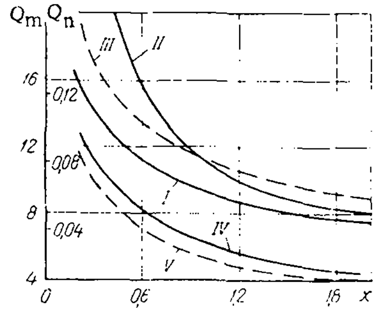

Fig. 2

Fig. 1. Temperature and concentration on film surface as function of x: I) temperature; II) concentration. The solid line is the exact solution, the dashed line is the approximate solution [1]. $x = (1/\mathrm{RePr}) \cdot (x'/\delta)$.

Fig. 2. Variation of dimensionless heat and mass fluxes along film: I) heat flux through solid wall (exact solution); II) heat flux through film surface (exact solution); III) heat flux through film (approximate solution [1]); IV) mass flux through film surface (exact solution); V) mass flux through film surface (approximate solution [1]).

$$Q_m = \frac{g_m \delta}{\rho D C_0}\Big|_{y=\delta} = \sum_n B_n \exp(-k_n^2 x)\, Y_n^{(2)\prime}(1) =$$
$$= -\sum_n B_n \exp(-k_n^2 x)\sqrt{\mathrm{Le}}\, k_n \sin\sqrt{\mathrm{Le}}\, k_n; \tag{23}$$

$$T_{av} = \frac{T_{av}}{T_0} = \frac{1}{\delta T_0}\int_0^\delta T\,dy = \frac{T_w}{T_0} -$$
$$- \sum_n \frac{A_n}{k_n}\exp(-k_n^2 x)(\cos k_n - 1); \tag{24}$$

$$C_{av} = \frac{C_{av}}{C_0} = \frac{1}{\delta C_0}\int_0^\delta C\,dy = \frac{dT_w - b}{C_0} + \frac{1}{\sqrt{\mathrm{Le}}} \times$$
$$\times \sum_n \frac{B_n}{k_n}\exp(-k_n^2 x)\sin k_n\sqrt{\mathrm{Le}}\,. \tag{25}$$

Thus, all the characteristics of the heat- and mass-transfer process within the film are determined by the four criteria Le, Re, Pr, and Ka and the parameters characterizing the initial state of the film (T_0, C_0, T_w, d, b).

Relations (14), (19)-(25) were used for specific calculations.

To obtain the numerical results in each case we tested the practical convergence of the respective expansions. For instance, to obtain the results shown in Figs. 1 and 2 we summed 30, 40, and 50 terms of the series. We found that all the characteristics calculated by using 40 and 50 terms agreed to within 10^{-5}.

For Le = 1 the roots of Eq. (14) are expressed in explicit form

$$k_n = \operatorname{arctg}\sqrt{-\frac{1}{\mathrm{Ka}}} + \pi(n-1),\quad n = 1, 2, \ldots .$$

For Le ≠ 1 Eq. (14) must be solved numerically. To simplify the solution we considered the case where $\sqrt{\mathrm{Le}}$ is a whole number. Then $f(k_n)$ is a periodic function with period π and, hence, for the determination of all the roots it is sufficient to solve Eq. (14) on the interval [0, π].

Figures 1 and 2 give some results of calculations for parameters characteristic of lithium bromide absorbers and compare the exact solutions with the approximate solutions that we obtained in [1].

NOTATION

$\bar{T}$, temperature, $T=(\bar{T}-T_w)/T_0$, dimensionless temperature; $\bar{C}$, concentration of absorbate in solution (mass fraction); T_w, wall temperature; d, b, constants determining state of saturation on liquid–vapor interface; $C=(\bar{C}-dT_w-b)/C_0$, dimensionless concentration; $y=y'/\delta$; $x=x'a/v\delta^2=(1/\mathrm{RePr})\cdot(x'/\delta)$; $\mathrm{Le}=a/D$, Lewis number; $\mathrm{Pr}=\nu/a$, Prandtl number; $\mathrm{Re}=V\delta/\nu$, Reynolds number; a, thermal diffusivity; D, diffusion coefficient; r_a, heat of absorption; c_p, specific heat; λ, thermal conductivity; ν, viscosity; ρ, density of solution; g_h, g_m, dimensional heat and mass fluxes, respectively; $\bar{T}_{av}$, $\bar{C}_{av}$, average values of temperature and concentration over cross section of film; Q_{hs}, Q_{hw}, dimensionless heat fluxes through film surface and solid wall; Q_m, dimensionless mass flux through film surface.

LITERATURE CITED

1. V. E. Nakoryakov and N. I. Grigor'eva, Inzh.-Fiz. Zh., 32, No. 3 (1977).

THEORY OF REACTION DIFFUSION FOR BODIES OF PLANE, CYLINDRICAL, AND SPHERICAL SYMMETRY

Yu. M. Grigor'ev, S. L. Kharatyan, Z S. Andrianova, A. N. Ivanova, and A G. Merzhanov

UDC 669:532.72

A solution of the nonstationary Stefan problem is presented for bodies of plane, cylindrical, and spherical symmetry in application to processes of diffusion interaction between metals and a gaseous oxidative medium.

The kinetics of metal interaction with gases is usually studied (see [1], for instance) by gravimetric (by the change in specimen weight), volumetric (by the quantity of absorbed gas), metallographic (by the periodic measurements of the thickness of the reaction-product films), and calorimetric (by the quantity of heat liberated by the reaction) methods. Hence, specimens of a different geometric shape (plates, wires, spherical particles, etc.) were used in tests. In this connection, it is interesting to analyze the question of the influence of the geometric shape of the specimens used on the regularity of reaction diffusion. Some results of such an analysis based on an assumption of a stationary distribution of the reagent concentration in the product film are contained in [2-4]. This question is analyzed in this paper in the general case of nonstationarity of mass transfer through the reaction-product film.

§ 1. Statement of the Problem

Within the framework of the classical theory of reaction diffusion [5], which is based on the assumption of the limiting role of transfer of the gaseous reagent through the reaction-product film, the process is described by a nonlinear Stefan problem, which has the following form for bodies of finite size but different geometric shape

$$\frac{\partial c}{\partial t}=D\left(\frac{\partial^2 c}{\partial x^2}+\frac{n}{x}\frac{\partial c}{\partial x}\right),\ r\leqslant x\leqslant R, \tag{1.1}$$

$$t=0\ \ c=c_1-\frac{c_1-c_2}{R-r}(R-x),\ \ R-r=\varepsilon\ll R, \tag{1.2}$$

$$x=R\quad c=c_1, \tag{1.3}$$

$$x=r\quad c=c_2, \tag{1.4}$$

$$c_2\frac{d(R-r)}{dt}=D\left.\frac{\partial c}{\partial x}\right|_{x=r}. \tag{1.5}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 33, No. 5, pp. 899-905, November, 1977. Original article submitted August 30, 1976.